Electrical Circuits

INTRODUCTION

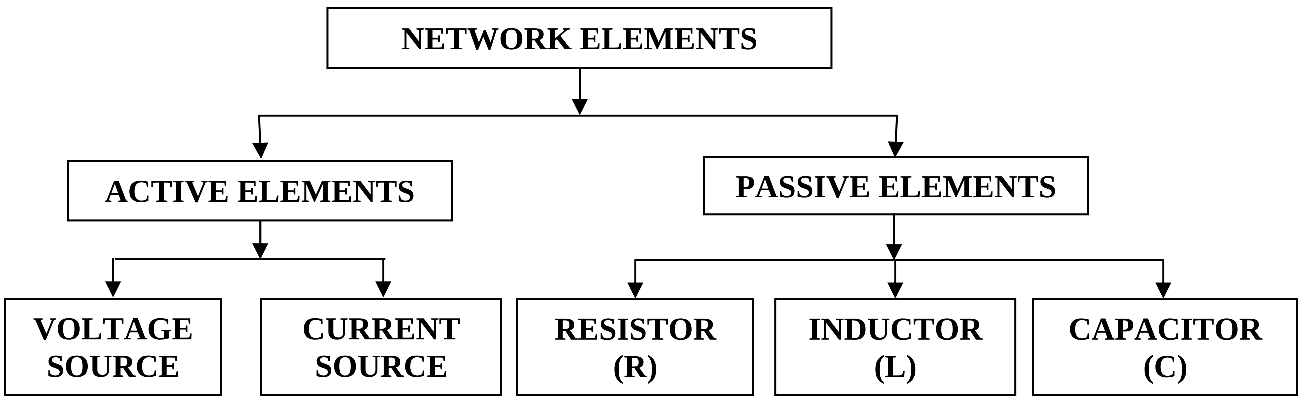
An electric circuit (or) network is the interconnection of energy sources and passive elements like Resistor 'R', Inductor 'L' and Capacitor 'C'. A physical electric circuit serves to transfer (or) transform energy. To explain any electric circuits (or) magnetic phenomena two concepts are employed i.e., **CIRCUIT CONCEPT & FIELD CONCEPT**.

A physical circuit (or) network is a system of interconnected network elements. When electric energy is supplied to a circuit it will respond in one (or) more of the following ways.

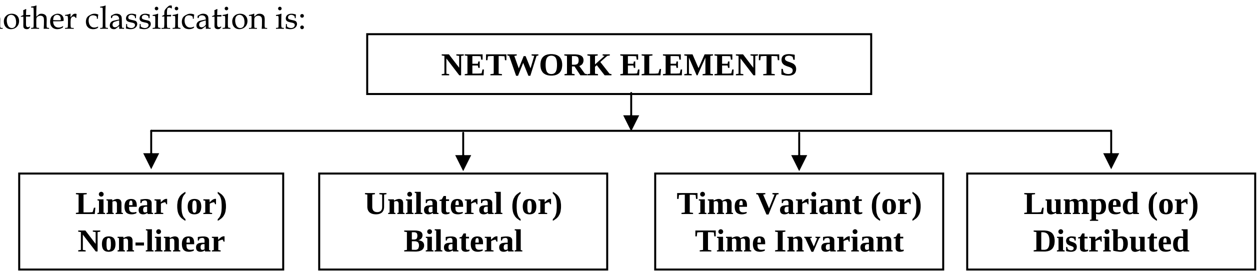
- ✓ If the energy is consumed (or) dissipated, then the circuit element is a pure resistor.
- ✓ If the energy is stored in magnetic field, then the element is a pure inductor.
- ✓ If the energy is stored in electric field, then the element is a pure capacitor.

CLASSIFICATION OF NETWORK ELEMENTS

The network elements can be classified as follows:



Another classification is:



In network analysis, given the circuit & excitation [Energy sources], we find the response [Current, Voltage] at any point in the network.

DEFINITIONS

CHARGE AND ENERGY:

The concept of charge is based on atomic theory. An atom has positive charges [Protons] in its nucleus and equal number of negative charges [electrons] surround the nucleus making the atom neutral, Removal of an electron leaves the atom positive charged and addition of an electron makes the atom negatively charged. The basic unit of charge is the charge on an electron. It's unit is coulombs.

Energy is defined as capacity to do work (or) energy can be defined as stored work (or) the energy is spent as work for transporting charge from one point to another in a circuit. The unit of energy is Joules. In electrical engineering one joule is defined as the energy required to transfers a power of one watt in one second to a load.

CURRENT AND VOLTAGE:

When a charge is transferred from one point in the circuit to another point it constitutes what is known as electric current. An electric current is defined as the time rate of flow of charge through a certain section. Its unit is Ampere. A current is said to be of one ampere when a charge of one coulomb flows through a section per second.

Mathematically, $$i = \frac{dq}{dt}$$

Where charge, 'q' is expressed in coulombs and time, 't' in seconds. Every charge will have a potential energy. The difference in potential energy between the charges is called potential difference. In electrical terminology the potential difference is called voltage. The potential difference indicates the amount of work done to move a charge from one place to another. The unit of voltage (or) potential difference is Volt. One volt is the potential difference between two points when one joule of energy is utilized in transferring one coulomb of charge from one point to the other.

Mathematically, $$v = \frac{dw}{dq}$$

POWER:

Power is the rate at which work is done (or) power is rate of energy transfer. The unit of power is Watt. One watt is defining as the power if the energy is transferred at the rate of one joule per second.

Mathematically, $P = \frac{dw}{dt} = \frac{dw}{dq} * \frac{dq}{dt} = v * i \Rightarrow dw = pdt \Rightarrow W(energy) = \int p\,dt$

Also, Power is product of voltage and current.

CIRCUIT ELEMENT:

Any individual circuit component [Resistor, Inductor, Capacitor] with two terminals, by which it can be connected to other electric components.

BRANCH:

A group of circuits, usually in series and with two terminals.

NETWORK AND CIRCUIT:

An electric network is any possible inter connection of electric circuit elements (or) branches.

An electric circuit is a closed energized network. A network is not necessary a circuit i.e.,

Example: T – Network.

ACTIVE AND PASSIVE ELEMENTS:

Active elements are the sources of energy. It may be a current source (or) voltage source, which supplies energy to the circuit.

An element which is not an energy source is a passive element. These elements either absorb or store energy i.e.,

Example: R, L, C

UNILATERAL AND BILATERAL ELEMENTS:

Unilateral elements offer low impedance for the flow of current in one direction and high impedance for the flow of current, in apposite direction. The current flowing through the element changes with the direction of current flow.

Example:- Vacuum Diode, Semi Conductor Device etc.,

A bilateral element offers same impedance for the flow current in either direction. The current flowing through the element does don't change with its direction.

Example:- Resistor, Inductor, Capacitor.

LINEAR AND NON-LINEAR ELEMENTS:

A linear element is one which is governed by a linear relationship [algebraic (or) differential equation] between the excitation and response. Otherwise, it is non-linear element. In the case of a linear element the value of the parameter [R (or) L (or) C] is independent of voltage and current through the element.

Example:-

- ✓ Resistor and Air cored inductor are linear elements.
- ✓ Germanium diode, Triode, Iron cored inductor are non-linear elements.

LUMPED AND DISTRIBUTED ELEMENTS:

A lumped element is one whose size is small compared to the wavelength corresponding to their normal frequency of operation; otherwise it is called a distributed element. The Kirchhoff's laws are only applicable to circuit with lumped elements.

Example:-

- ✓ Resistor, Inductor, Capacitor is lumped elements.
- ✓ A Long transmission lines are a distributed parameter network.

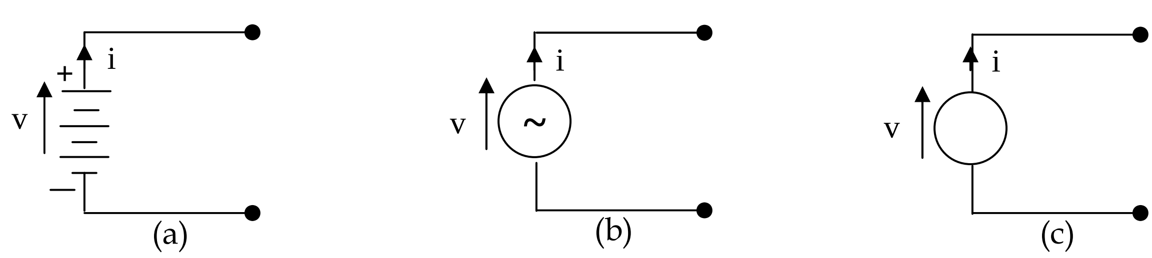
TIME VARIENT AND TIME INVARIANT ELEMENTS:

If the parameters of network elements do not vary with time, they are called time invariant elements, other wise they are called time variant.

VOLTAGE & CURRENT SOURCES

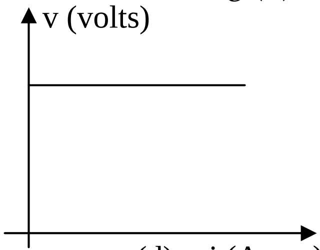
The mostly used energy sources are [in practice] Battery, Signal generators, Thermo couples, Solar cells and other types of sources. These sources will provide an electro-motive force (e.m.f) and they are responsible for a potential drop between the points to which they are connected and hence produce a certain amount of current flow.

VOLTAGE SOURCES:

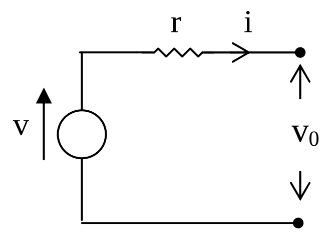


The voltage source is one which establishes a potential difference across its terminals. If this is constant then that voltage source is an ideal one [for any current drawn]. The symbolic representation of a direct current and alternating current, ideal voltage sources are represented in fig.'s (a) & (b) respectively.

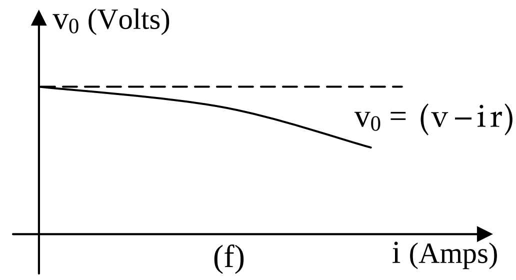
In fig.(c) shows the general symbol for voltage source which may be a direct voltage, alternating voltage (or) any other time varying quantity. The voltage & current relationship of an ideal voltage sources as shown in fig.(d).



(d)



(e)

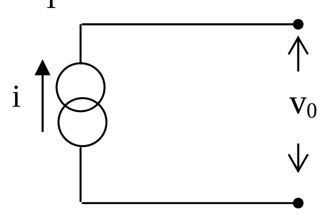


(f)

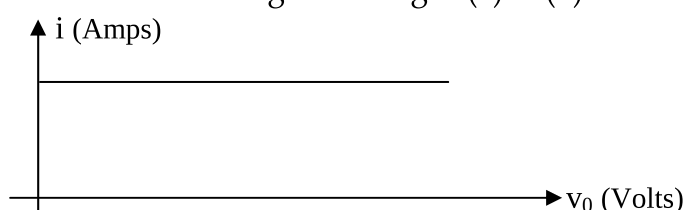
In Non-ideal voltage source is the one which is with varying voltage based on the current drawn from the source. This can be modeled as shown in fig.(e). The output characteristics of the non-ideal sources is depected in fig.(f).

CURRENT SOURCES:

An ideal source is one which maintains a constant terminals current independent of the load. The model representation and its output characteristics are given in fig.'s (a) & (b).

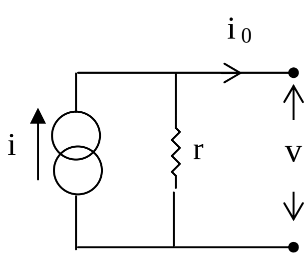


(a)

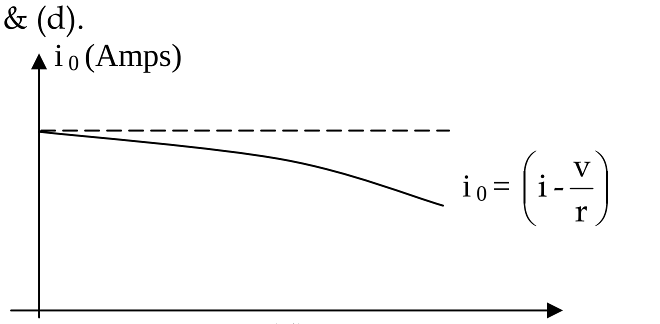


(b)

A non-ideal current source is the one which cannot supply a current that is independent of the load to which the source is connected. The model of a non-ideal current source & its output characteristics are given in fig.'s (c) & (d).



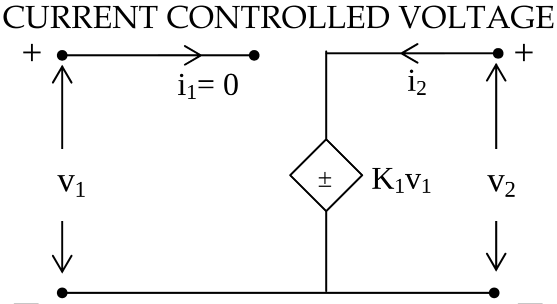
(c)



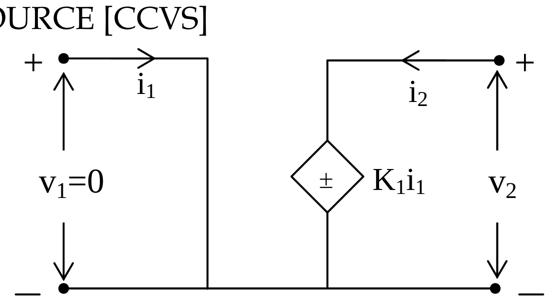
(d) v (Volts)

The sources explained above are called independent sources. The independent sources are characterized by a defined terminal voltage & current as the case may be. The dependent sources are that, whose output is not fixed and is dependent on the value of the input. These sources are widely used in electronic circuit. These are basically classified as,

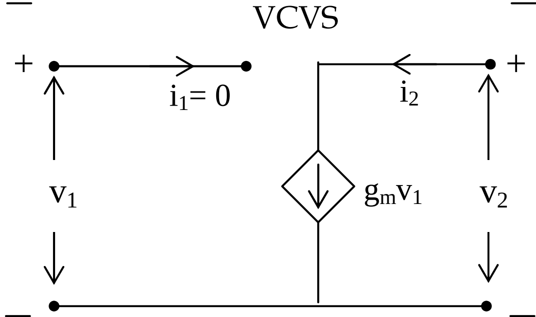
- ✓ VOLTAGE CONTROLLED VOLTAGE SOURCE [VCVS]
- ✓ CURRENT CONTROLLED CURRENT SOURCE [CCCS]
- ✓ VOLTAGE CONTROLLED CURRENT SOURCE [VCCS]
- ✓ CURRENT CONTROLLED VOLTAGE SOURCE [CCVS]



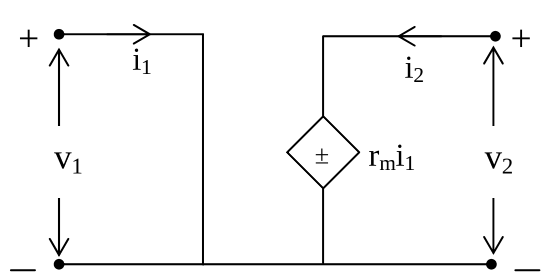
VCVS



CCCS



VCCS



CCVS

CIRCUIT COMPONENTS

The circuit parameters considered in the section are assumed to be **Lumped, Linear, Passive & Bilateral**. According to the circuit representation of the parameters, namely resistance, inductance and capacitance is discussed here only along with their voltage & current relations using **Ohm's Law**. The behavior of these components for different types of signals is presented in this section.

OHM'S LAW

As the rate of flow of water in a pipe is directly proportional to the effective pressure [difference of pressure at two ends] and inversely proportional to the frictional resistance, similarly, the current flowing through a conductor is directly proportional to the potential difference across the ends of the conductor and inversely proportional to the conductor resistance. This relationship was discovered by **GEORG SIMON OHM** and hence is known as Ohm's Law. It is denoted by 'Ω'(or) 'Ohm'.

If 'I' is the current flowing through a conductor of resistance 'R' across which a potential difference 'V' is applied, then according to ohm's law.

$$I \propto V \;\; \& \;\; I \propto \frac{1}{R} \text{ (or) } I \propto \frac{V}{R} \text{ (or) } I = \frac{V}{R}$$

Where I→ is current in Amperes, V→ is voltage in Volts & R→ is resistance in Ohms.

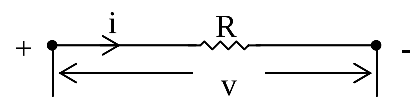
NOTE : "Ohm's law cannot be applied to circuits consisting of electronic valves (or) transistors because these elements are not bilateral and also non-linear elements".

RESISTANCE

In most of the materials, the current that flow depends on the potential difference that exist between two points of interest, hence the resistance to the flow of current is proportional to this potential difference. If 'R' is the resistance of the conducting material measure in Ohm's and 'v', 'i' are the instantaneous quantities of voltage and current as shown in fig..

Using Ohm's law the formula is

$$v = iR \qquad ---(1)$$



Eq.(1) may be also written as,

$$i = \frac{v}{R} = Gv \qquad ---(2)$$

Where 'G' is the reciprocal of the resistance called conductance, measured is Siemen's (or) mho's [Symbol '℧' (or) 'S']

The arrow direction of current indicates from positive terminal to negative terminal of the potential difference as shown in fig..

RESISTANCE IN SERIES:

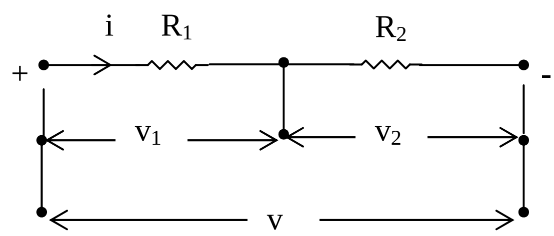
If 'R_1' & 'R_2' are two resistors connected in series as shown in fig., then to maintain the continuity of charge movement, the same current must flow through 'R_1' and 'R_2'.

The total voltage 'v' is given by

$$v = v_1 + v_2 = i(R_1 + R_2)$$

$$\because \; v_1 = iR_1 \;\; \& \;\; v_2 = iR_2$$

$$\Rightarrow v = iR \qquad ---(1)$$



Where $R = R_1 + R_2 \;\; \& \;\; v_2 = \frac{v*R_2}{R_1+R_2}; \quad v_1 = \frac{v*R_1}{R_1+R_2}$

If 'n' resistors are connected in series then the effective resistance 'R' is given by,

$$R = \sum_{i=1}^{n} R_i = R_1 + R_2 + \ldots\ldots\ldots\ldots + R_n \qquad ---(2)$$

RESISTANCE IN PARALLEL:

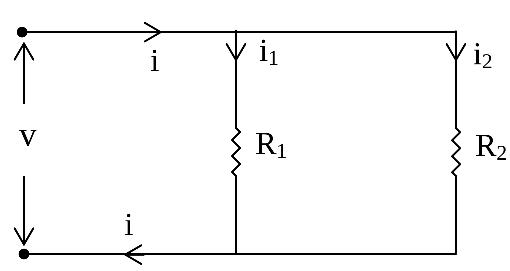
If 'R_1' and 'R_2' are connected in parallel as shown in fig., then the current flowing in each branch are 'i_1' and 'i_2'. The total current is given as,

$$i = i_1 + i_2 \qquad ---(1)$$

$$\because i = \frac{v}{R}; \quad i_1 = \frac{v}{R_1}; \quad i_2 = \frac{v}{R_2}$$

$$\therefore \frac{v}{R} = \frac{v}{R_1} + \frac{v}{R_2} = v\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

$$\Rightarrow \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \qquad ---(2)$$



Where R→ is the effective resistance of the circuit, is given as

$$R = \frac{R_1 R_2}{R_1 + R_2} \qquad ---(3) \qquad \text{and} \qquad i_1 = \frac{i*R_2}{R_1+R_2}; \qquad i_2 = \frac{i*R_1}{R_1+R_2}$$

If these are 'n' resistance connected in parallel, then

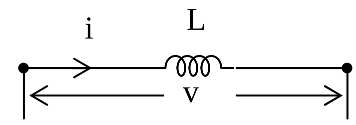
$$\frac{1}{R}=\sum_{i=1}^{n}\frac{1}{R_i}=\frac{1}{R_1}+\frac{1}{R_2}+.............+\frac{1}{R_n} \qquad ---(4)$$

The energy stored in a resistor,

$$\left.\begin{aligned} W&=\int v\ i\ dt=\int iR\ i\ dt=R\int i^2\ dt \\ \text{(or)}\quad W&=\int v\ i\ dt=\int v\ \frac{v}{R}\ dt=\frac{1}{R}\int v^2\ dt \end{aligned}\right\}\ \text{Joules} \qquad ---(6)$$

INDUCTANCE

If a current flows through a conductor, it causes a magnetic flux to be setup in the medium surrounding the conductor. In the coil, that is wound by using a conductor will strengthen the magnetic field. If 'N' is the number of turns and 'ϕ' is the flux linkage associated with a current 'i' flowing in the coil then $\phi = Li$. Where L$\rightarrow$ is the self inductance of the coil and is measured in Henries [Symbol 'H']. If the current in the coil is changing with time then an e.m.f is inducted in the coil. This is **Faraday's Law** and the voltage induced is given by



$$e=-\frac{N\,d\phi}{dt}\cong\frac{-L\,di}{dt} \qquad ---(1)$$

If inductance is constant then, $e=-\frac{di}{dt}$. The negative sign indicates the induced voltage in the coil opposes the potential difference which causes the changing current 'i' in the coil. This is due to **Lenz's Law**. The representation of the inductance as shown in fig. and 'v' is the potential difference then,

$$v=-e=L\frac{di}{dt}$$

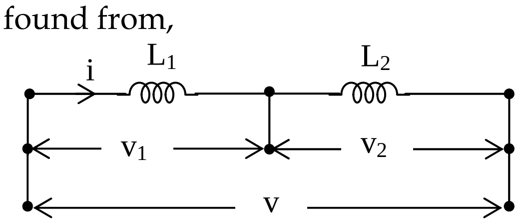
Assuming $v=0$ for $t<0$ then, $i=\frac{1}{L}\int_0^t v\ dt+i(o)$

Where $i(0)$ is the initial current in the coil at $t=0$. In the time interval $t=0^-$ to $t=0^+$ the current $i(0^+)=i(0^-)$. For a finite value of 'v' the current cannot change instantaneously.

NOTE : "In case of Inductance, current flowing through it cannot be changed instantaneously".

INDUCTANCE IN SERIES:

The fig. shows two inductors are connected in series. Assuming that the magnetic flux in each inductor do not interact, then the effective inductance is found from,



$$\begin{aligned} v&=v_1+v_2 \\ &=L_1\frac{di}{dt}+L_2\frac{di}{dt}=(L_1+L_2)\frac{di}{dt} \\ &=L\frac{di}{dt} \end{aligned}$$

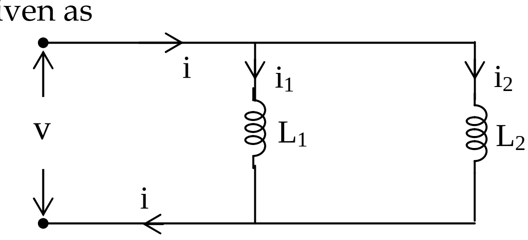
$\therefore$ L = L_1 +L_2 $\quad ---(2)$

If 'n' number of inductors are in series then the effective inductance,

$$L=\sum_{i=1}^{n}L_i=L_1+L_2+.............+L_n \qquad ---(3)$$

INDUCTANCE IN PARALLEL:

Two inductors are in parallely connected and assuming no interaction between the two inductors in the circuit are as shown in fig.. The total current 'i' is given as



$$i=i_1+i_2$$

$$\Rightarrow \frac{1}{L}\int v\ dt=\frac{1}{L_1}\int v\ dt+\frac{1}{L_2}\int v\ dt$$

$$\therefore \quad \frac{1}{L}=\frac{1}{L_1}+\frac{1}{L_2} \qquad ---(4)$$

If 'n' number of inductors are in parallel then the effective inductance,

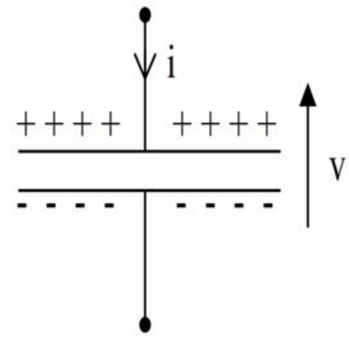
$$\frac{1}{L_1}=\sum_{i=1}^{n}\frac{1}{L_i}=\frac{1}{L_1}+\frac{1}{L_2}+.............+\frac{1}{L_n} \qquad ---(5)$$

The energy stored in a inductor,

$$W=\int v\ i\ dt=\int L\frac{di}{dt}\ i\ dt=L\int i\ di=\frac{Li^2}{2}\ \text{Joules} \qquad ---(6)$$

CAPACITANCE

A pair of conducting bodies separated by an insulating material forms a capacitor (or) Condenser.



If a potential difference is established between the terminals of a capacitor there is a current flow as shown in figure. The negatively charged electrons cannot travels in the space between the plates, and move in the direction opposite to that of the conventional current. The electrons leaving in the upper plate will create a positive charge and will accumulate equal negative charge on the lower plate. This phenomenon will continue until the force between the two plates is equal and opposite to the force. By the electrons due to the applied voltage at the point the current is zero and the capacitor is said to be charged.
If this source of energy is removed the charge is stored & will be redistributed, if it is connected across some resistance and permit the electrons to move to have a continuous current.
The capacitance of a two parallel plate capacitor is given by,

$$C = \frac{\in_0 \in_r A}{d} \quad \text{Farads} \quad ---(1)$$

Where, A→ Area of cross section in m^2
d → Distance between the plates in m
$\in_0$→ Permittivity of free space &
$\in_r$→ Relative permittivity of the media between the two plates.

The capacitance will be more if '$\in_r$' is more for the same configuration of the plates. The relationship between the charge 'q' stored on the plate of the capacitor and the applied voltage 'v' is given by

$$q = Cv \quad ---(2)$$

Where 'C' stands for the capacitance and the units are farads [Symbol 'F']. If the charge & voltage are time variant then,

$$\frac{dq}{dt} = C\frac{dv}{dt} \quad ---(3)$$

Note: 1. If 'C' is assumed to be constant,

$$\frac{dq}{dt} = C\frac{dv}{dt} + v\frac{dc}{dt} \quad ---(4)$$

2. If 'C' is also varying with time and the current through the capacitor is equal to,

$$i = C\frac{dv}{dt} \quad ---(5)$$

The expression for the voltage across the capacitor at any time, 't' is given by

$$\frac{dv}{dt} = \frac{i}{C} \quad ---(6)$$

Integrating the above equation, we have

$$v = \int_0^t \frac{i}{c} dt + v(o) = \frac{1}{c}\int_0^t i\,dt + v(o) \quad ---(7)$$

Where $v(0)$ → is the initial voltage on the capacitor at time $t = 0$. If $v(0^+)$ is the voltage at $t = (0^+)$ & $v(0^-)$ is at time $t = (0^-)$ then,

$$v(o^+) = \frac{1}{c}\int_{0^-}^{0^+} i\,dt + v(0^-) \quad ---(8)$$

As for as the charging current is finite then

$$\frac{1}{c}\int_{0^-}^{0^+} i\,dt = 0 \quad ---(9)$$

From equation (8) & (9), we have

$$v(0^+) = v(0^-) \quad ---(10)$$

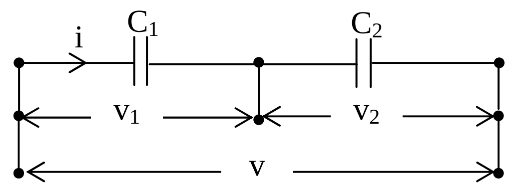
Equation (10) says that as far as the current is infinite the voltage across the capacitor cannot change instantaneously. When potential 'v' is applied suddenly to an uncharged capacitor 'c' during a small interval $t = (0^-)$ to $t = (0^+)$ and the requisite charge 'dq' is deposited in this small time then, $i = \frac{dq}{dt} \cong \infty$.
The current has an infinite value at $t = 0$ & zero at all times. Such a function is called impulse function [having an infinity height & zero width].
NOTE : "In case of a Capacitor, the voltage across it cannot be changed instantaneously".
The energy stored in a capacitor is,

$$W=\int vi\,dt=\int vc\frac{dv}{dt}dt=c\int v\,dt=\frac{cv^2}{2}\quad \text{Joules}\quad ---(11)$$

CAPACITANCE IN SERIES:

The connection of two capacitors 'C_1' and 'C_2' is shown in figure. From the figure,

$$v_1=\frac{1}{C_1}\int i\,dt\quad ---(1)\ \&\ v_2=\frac{1}{C_2}\int i\,dt\quad ---(2)$$



Here, $v=v_1+v_2=\left[\frac{1}{C_1}+\frac{1}{C_2}\right]\int i\,dt$

The effective capacitance, $C=\frac{C_1C_2}{C_1+C_2}\quad ---(3)$

If 'n' capacitors are connected in series then the effective capacitance is given by

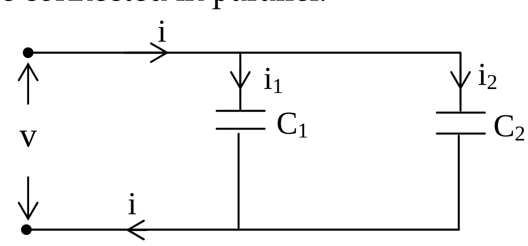
$$\frac{1}{C}=\sum_{i=1}^{n}\frac{1}{C_i}=\frac{1}{C_1}+\frac{1}{C_2}+.............+\frac{1}{C_n}\quad ---(4)$$

CAPACITANCE IN PARALLEL:

In practical systems the value of capacitance may be of order of Micro farads to Pico farads. When a higher value of capacitance is needed it is obtained by connecting in parallel [As the unit of farad is extremely, Large]. The figure shows, two capacitors are connected in parallel.

The total current supplied is,

$$i=i_1+i_2\quad ---(1)$$



i.e., $C\frac{dv}{dt}=C_1\frac{dv}{dt}+C_2\frac{dv}{dt}=(C_1+C_2)\frac{dv}{dt}\quad ---(2)$

If 'C' is the effective capacitance, then

$$C = C_1 + C_2\quad --- (3)$$

If 'n' capacitances of the capacitors are connected in parallel then the effective capacitance will be,

$$C=\sum_{i=1}^{n}C_i=C_1+C_2+\cdots\cdots+C_n\quad ---(4)$$

KIRCHHOFF'S LAW

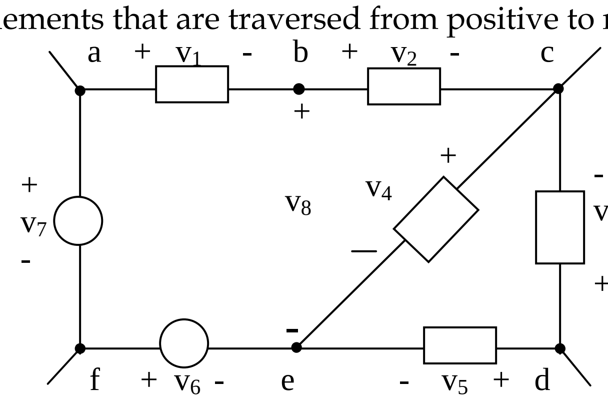
KIRCHHOFF'S VOLTAGE LAW [K.V.L]:

At any instant of time, the Algebraic sum of all voltages around a closed path is zero.

Mathematically, $\sum_{\text{closed path}} v(t)=0$

Closed Path: The closed path is to be traced in an arbitrary specified direction, which may be clockwise (or) anti clockwise direction for traversing the closed path.

The voltage across the elements that are traversed from negative to positive are taken positive & the voltage across the elements that are traversed from positive to negative are taken as negative.



From fig., the voltage across the elements, in the network, KVL around mesh abcefa,

$$V_{ab}+V_{bc}+V_{ce}+V_{ef}+V_{fa}=0\Rightarrow -V_1-V_2-V_4+V_6+V_7\Rightarrow V_6+V_7=V_1+V_2+V_4$$

Also, Sum of Rising voltages = Sum of Dropping Voltages. It is to be noted that while writing KVL equations, it is necessary to have a physically closed connection of elements. It should be a closed path. It may not be closed circuit. For example, there is no element connected between 'b' & 'e'. By applying KVL, bcdeb

$$V_{bc}+V_{cd}+V_{de}+V_{eb}=0\Rightarrow -V_2+V_3-V_5+V_8=0\Rightarrow V_3+V_8=V_2+V_5$$

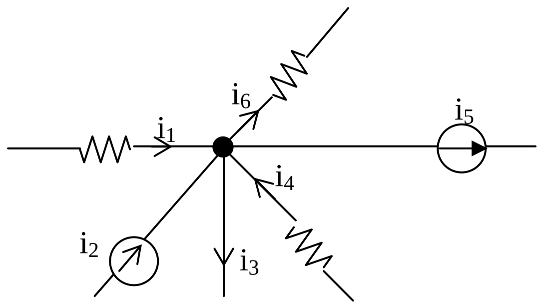
Since, voltage is energy (or) work per unit charge, KVL is an alternative method of stating the law of conservation of energy.

KIRCHHOFF'S CURRENT LAW [KCL]

At any instant of time, the algebraic sum of currents at a node (or) point (or) junction is zero.

Mathematically, $\sum_{\text{node}} i(t)=0$

If the currents entering a node are assigned positive sign, then the currents leaving the node will be assigned negative sign (or) vice versa. The choice of the sign convention is arbitrary, but once a sign convention is chosen, it should not be changed with respect to any particular node while writing KCL equation. In applying KCL let us adopt the sign convention that the incoming currents are positive & the outgoing currents are negative.



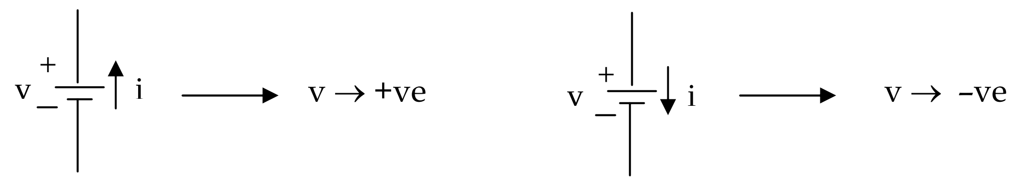
Consider the network as shown in fig. and currents i_1, i_2 & i_4 are entering the nodes are assigned positive and currents i_3, i_5 & i_6 are leaving the node are assigned negative sign. Applying KCL, at node we get,

$$i_1 + i_2 - i_3 + i_4 - i_5 - i_6 = 0 \Rightarrow i_1 + i_2 + i_4 = i_3 + i_5 + i_6$$

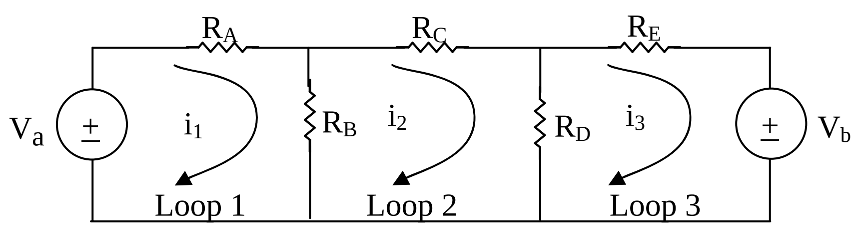
Also, Sum of incoming current = Sum of outgoing currents.

KCL is based on the conservation of charge at anode. According to the law of conservation of charge, the charge flowing into a node is equal to the net charge flowing out of the node. $\left[\because\ i = \frac{dq}{dt}\right]$

NOTE:



MESH ANALYSIS



Apply KVL at

Loop1, $V_a - R_A i_1 - R_B(i_1 - i_2) = 0 \Rightarrow (R_A + R_B)i_1 - R_B i_2 = V_a \quad ---(1)$

Loop2, $-R_B(i_2 - i_1) - R_C i_2 - R_D(i_2 - i_3) = 0 \Rightarrow -R_B i_1 + (R_B + R_C + R_D)i_2 - R_D i_3 = 0 \quad ---(2)$

Loop3, $-R_D(i_3 - i_2) - R_E i_3 - V_b = 0 \Rightarrow -R_D i_2 + (R_D + R_E)i_3 = -V_b \quad ---(3)$

Putting in matrix form,

$$\begin{bmatrix} R_A + R_B & -R_B & 0 \\ -R_B & R_B + R_C + R_D & -R_D \\ 0 & -R_D & R_D + R_E \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ -V_b \end{bmatrix}$$

$$\text{Where } \Delta = \begin{bmatrix} R_A + R_B & -R_B & 0 \\ -R_B & R_B + R_C + R_D & -R_D \\ 0 & -R_D & R_D + R_E \end{bmatrix}$$

Using crammer's rule, calculate the loop current i_1, i_2 & i_3.

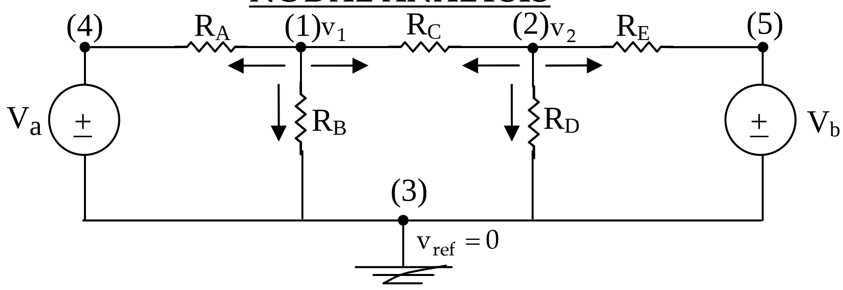
$$\Delta_1 = \begin{bmatrix} V_a & -R_B & 0 \\ 0 & R_B + R_C + R_D & -R_D \\ -V_b & -R_D & R_D + R_E \end{bmatrix} \qquad i_1 = \frac{\Delta_1}{\Delta}$$

$$\Delta_2 = \begin{bmatrix} R_A + R_B & V_a & 0 \\ -R_B & 0 & -R_D \\ 0 & -V_b & R_D + R_E \end{bmatrix} \qquad i_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta_3 = \begin{bmatrix} R_A + R_B & -R_B & V_a \\ -R_B & R_B + R_C + R_D & 0 \\ 0 & -R_D & -V_B \end{bmatrix} \qquad i_3 = \frac{\Delta_3}{\Delta}$$

NOTE: n → Loops: n → equations / n x n order matrix

NODAL ANALYSIS



Nodes 4, 5 → Simple nodes & 1, 2, 3 → Principle nodes

A Principle node means three (or) more branches meeting at a point. Select one reference node as a principle node i.e., reference node voltage v_{ref} is zero. By applying KCL,

At node (1), $\dfrac{v_1 - V_a}{R_A} + \dfrac{v_1}{R_B} + \dfrac{v_1 - v_2}{R_C} = 0 \Rightarrow v_1\left[\dfrac{1}{R_A} + \dfrac{1}{R_B} + \dfrac{1}{R_C}\right] - v_2\left[\dfrac{1}{R_C}\right] = V_a\left[\dfrac{1}{R_A}\right] \quad ---(1)$

At node (2), $\dfrac{v_2 - v_1}{R_C} + \dfrac{v_2}{R_D} + \dfrac{v_2 - V_b}{R_E} = 0 \Rightarrow -v_1\left[\dfrac{1}{R_C}\right] + v_2\left[\dfrac{1}{R_C} + \dfrac{1}{R_D} + \dfrac{1}{R_E}\right] = V_b\left[\dfrac{1}{R_E}\right] \quad ---(2)$

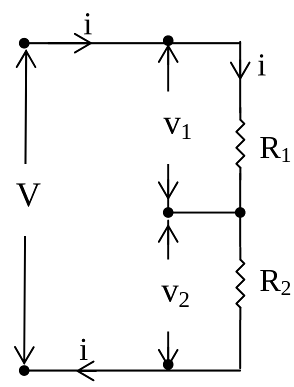
Putting in matrix form

$$\begin{bmatrix} \dfrac{1}{R_A} + \dfrac{1}{R_B} + \dfrac{1}{R_C} & -\dfrac{1}{R_C} \\ -\dfrac{1}{R_C} & \dfrac{1}{R_C} + \dfrac{1}{R_D} + \dfrac{1}{R_E} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} V_a/R_A \\ V_b/R_E \end{bmatrix}$$

NOTE: $n \rightarrow$ Loops: $n-1 \rightarrow$ equations & $n-1 \times n-1$ order matrix

VOLTAGE DIVISION

A series circuit acts as a voltage divider as the voltage divides in all elements in a series circuit. In fig., the voltage 'V' has been divided into 'v_1' and 'v_2' in two elements with resistances 'R_1' and 'R_2' while 'V' represents the applied voltage and 'i' the current flowing through the two resistances 'R_1' and 'R_2'. **In case of series connection the total current must be same**.



Obviously, $v_1 = iR_1$ & $V_2 = iR_2$

Let 'R' represents the total resistance and is given by

$R = R_1 + R_2$

$i = \dfrac{V}{R} = \dfrac{V}{(R_1 + R_2)}$

Drop across R_1, $v_1 = iR_1 = \dfrac{VR_1}{(R_1 + R_2)}$

Drop across R_2, $v_2 = iR_2 = \dfrac{VR_2}{(R_1 + R_2)}$

Thus, the voltage across any resistor in a series circuit is equal to the value of that total applied voltage times the same resistor value divided by the total two resistors connected in series.

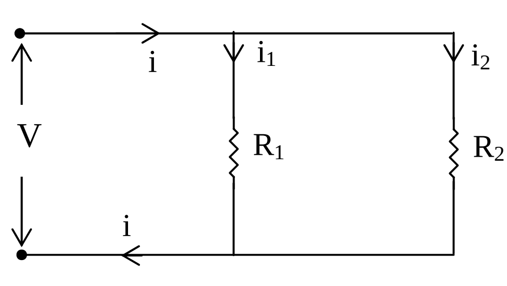
CURRENT DIVISION

A parallel circuit acts as a current divider as the current divides in all branches in a parallel circuit. In fig., the current 'i' has been divided into 'i_1' and 'i_2' in two parallel branches with resistances 'R_1' and 'R_2' while 'V' represents the drop across 'R_1' (or) 'R_2'. **In case of parallel connection the total applied voltage across each branch must be same**.

Obviously, $i_1 = \dfrac{V}{R_1}$ and $i_2 = \dfrac{V}{R_2}$

Let 'R' represents the total resistance and is given by $R = \dfrac{R_1 R_2}{R_1 + R_2}$

Also, $i = \dfrac{V}{R} = \dfrac{V}{R_1 R_2}(R_1 + R_2) \quad ---(1)$



But $V = i_1 R_1 = i_2 R_2$

Put, $V = i_1 R_1$ in eq.(1)

$i = \dfrac{i_1 R_1 (R_1 + R_2)}{R_1 R_2} = \dfrac{i_1 (R_1 + R_2)}{R_2} \Rightarrow i_1 = \dfrac{iR_2}{(R_1 + R_2)}$

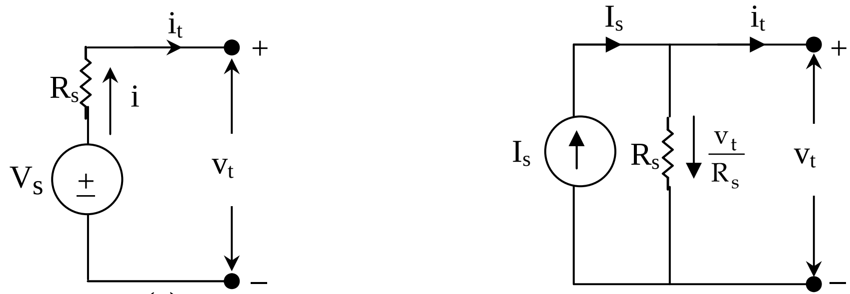
Put, $V = i_2 R_2$ in eq.(1)

$i = \dfrac{i_2 R_2 (R_1 + R_2)}{R_1 R_2} = \dfrac{i_2 (R_1 + R_2)}{R_1} \Rightarrow i_2 = \dfrac{iR_1}{(R_1 + R_2)}$

Thus, the current in any branch resistor in a parallel circuit is equal to the value of that total current times the opposite resistor value divided by the total two resistors are connected in parallel.

SOURCE TRANSFORMATION

Practical voltage source has a resistance in series with it and a practical current source has a resistance across it. When analyzing networks, with a view to reducing the complexity of the networks, it sometimes becomes necessary to replace voltage sources by current sources and vice–versa. This can be readily done if an equivalent can be established between a voltage source and a current source.



(a) (b)

Consider the practical voltage source and a current source as shown in fig.'s(a) & (b). For both sources, the voltage at the terminals is 'v_t', and the current supplied to an external network connected to these terminals is 'i_t'.

For the voltage source, we have

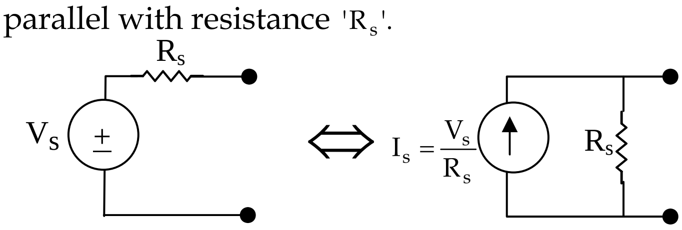
$$v_t = V_s - R_s i = V_s - R_s i_t \qquad (\because i = i_t)$$

$$\Rightarrow i_t = \frac{v_t - V_s}{-R_s} = \frac{V_s - v_t}{R_s} = \frac{V_s}{R_s} - \frac{v_t}{R_s} \quad ---(1)$$

For the current source, we have

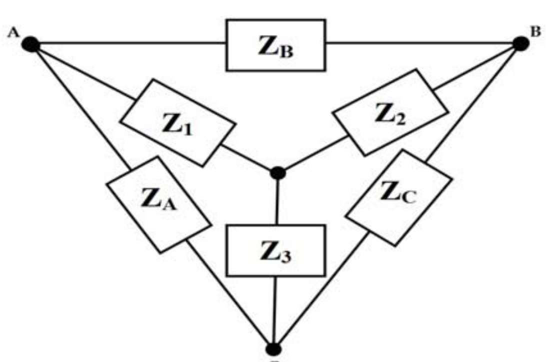
$$I_s = i_t + \frac{v_t}{R_s} \Rightarrow i_t = I_s - \frac{v_t}{R_s} \quad ---(2)$$

Comparing eq.'s(1) & (2), a voltage source 'V_s' with resistance 'R_s' in series with its equivalent to a current source $I_s = \frac{V_s}{R_s}$, in parallel with resistance 'R_s'.



DELTA TO STAR TRANSFORMATION

The derivation of delta to star conversion is follows:



$$Z_{AB} = Z_1 + Z_2, Z_{BC} = Z_2 + Z_3 \;\&\; Z_{CA} = Z_3 + Z_1$$

$$Z_{AB} + Z_{BC} + Z_{CA} = Z_1 + Z_2 + Z_2 + Z_3 + Z_3 + Z_1$$

$$= 3(Z_1 + Z_2 + Z_3) \quad ---(1)$$

$$Z_{AB} = Z_B \,||\, (Z_A + Z_C) = \frac{Z_B(Z_A + Z_C)}{(Z_A + Z_B + Z_C)} = \frac{Z_A Z_B + Z_B Z_C}{(Z_A + Z_B + Z_C)}$$

$$Z_{BC} = Z_C \,||\, (Z_A + Z_B) = \frac{Z_C(Z_B + Z_A)}{(Z_A + Z_B + Z_C)} = \frac{Z_B Z_C + Z_C Z_A}{(Z_A + Z_B + Z_C)}$$

$$Z_{CA} = Z_A \,||\, (Z_C + Z_B) = \frac{Z_A(Z_C + Z_B)}{(Z_A + Z_B + Z_C)} = \frac{Z_C Z_A + Z_A Z_B}{(Z_A + Z_B + Z_C)}$$

$$Z_{AB} + Z_{BC} + Z_{CA} = \frac{Z_A Z_B + Z_B Z_C}{(Z_A + Z_B + Z_C)} + \frac{Z_B Z_C + Z_C Z_A}{(Z_A + Z_B + Z_C)} + \frac{Z_C Z_A + Z_A Z_B}{(Z_A + Z_B + Z_C)}$$

$$= \frac{3(Z_A Z_B + Z_B Z_C + Z_C Z_A)}{(Z_A + Z_B + Z_C)} \quad ---(2)$$

Comparing eq.'s (1) & (2), we have,

$$3(Z_1 + Z_2 + Z_3) = \frac{3(Z_A Z_B + Z_B Z_C + Z_C Z_A)}{(Z_A + Z_B + Z_C)}$$

$$\Rightarrow (Z_1 + Z_2 + Z_3) = \frac{(Z_A Z_B + Z_B Z_C + Z_C Z_A)}{(Z_A + Z_B + Z_C)} \quad ---(3)$$

Form eq.(3),

$$(Z_1 + Z_2 + Z_3) = \frac{(Z_A Z_B + Z_B Z_C + Z_C Z_A)}{(Z_A + Z_B + Z_C)}$$

$$\Rightarrow (Z_1 + Z_{BC}) = \frac{(Z_A Z_B + Z_B Z_C + Z_C Z_A)}{(Z_A + Z_B + Z_C)} \qquad (\because \quad Z_2 + Z_3 = Z_{BC})$$

$$\Rightarrow Z_1 = \frac{(Z_A Z_B + Z_B Z_C + Z_C Z_A)}{(Z_A + Z_B + Z_C)} - Z_{BC} = \frac{(Z_A Z_B + Z_B Z_C + Z_C Z_A)}{(Z_A + Z_B + Z_C)} - Z_C \,||\, (Z_A + Z_B)$$

$$\Rightarrow Z_1 = \frac{Z_A Z_B}{(Z_A + Z_B + Z_C)} \qquad ---(4)$$

Form eq.(3),

$$(Z_1 + Z_2 + Z_3) = \frac{(Z_A Z_B + Z_B Z_C + Z_C Z_A)}{(Z_A + Z_B + Z_C)}$$

$$\Rightarrow (Z_2 + Z_{CA}) = \frac{(Z_A Z_B + Z_B Z_C + Z_C Z_A)}{(Z_A + Z_B + Z_C)} \qquad (\because \quad Z_3 + Z_1 = Z_{CA})$$

$$\Rightarrow Z_2 = \frac{(Z_A Z_B + Z_B Z_C + Z_C Z_A)}{(Z_A + Z_B + Z_C)} - Z_{CA} = \frac{(Z_A Z_B + Z_B Z_C + Z_C Z_A)}{(Z_A + Z_B + Z_C)} - Z_A \,||\, (Z_C + Z_B)$$

$$\Rightarrow Z_2 = \frac{Z_B Z_C}{(Z_A + Z_B + Z_C)} \qquad ---(5)$$

Form eq.(3),

$$(Z_1 + Z_2 + Z_3) = \frac{(Z_A Z_B + Z_B Z_C + Z_C Z_A)}{(Z_A + Z_B + Z_C)}$$

$$\Rightarrow (Z_3 + Z_{AB}) = \frac{(Z_A Z_B + Z_B Z_C + Z_C Z_A)}{(Z_A + Z_B + Z_C)} \qquad (\because \quad Z_1 + Z_2 = Z_{AB})$$

$$\Rightarrow Z_3 = \frac{(Z_A Z_B + Z_B Z_C + Z_C Z_A)}{(Z_A + Z_B + Z_C)} - Z_{AB} = \frac{(Z_A Z_B + Z_B Z_C + Z_C Z_A)}{(Z_A + Z_B + Z_C)} - Z_B \,||\, (Z_A + Z_C)$$

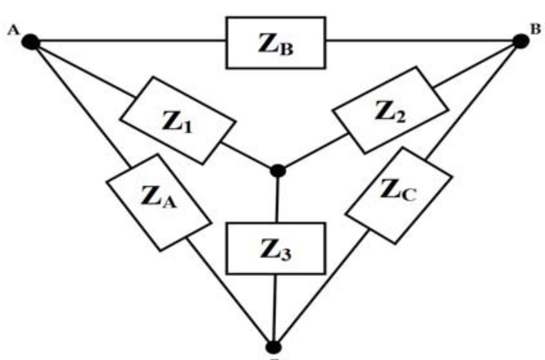
$$\Rightarrow Z_3 = \frac{Z_C Z_A}{(Z_A + Z_B + Z_C)} \qquad ---(6)$$

$$Z_1 = \frac{Z_A Z_B}{Z_A + Z_B + Z_C}, \; Z_2 = \frac{Z_B Z_C}{Z_A + Z_B + Z_C} \;\&\; Z_3 = \frac{Z_C Z_A}{Z_A + Z_B + Z_C}$$

Note: "The Star element is equal to the product of adjacent Delta elements divided by sum of Delta elements".

STAR TO DELTA TRANSFORMATION

The derivation of star to delta conversion is follows:



We know that, the delta to star conversion is,

$$Z_1 = \frac{Z_A Z_B}{Z_A + Z_B + Z_C}, \; Z_2 = \frac{Z_B Z_C}{Z_A + Z_B + Z_C} \;\&\; Z_3 = \frac{Z_C Z_A}{Z_A + Z_B + Z_C}$$

$$Z_1 Z_2 = \frac{Z_A Z_B}{Z_A + Z_B + Z_C} \times \frac{Z_B Z_C}{Z_A + Z_B + Z_C} = \frac{Z_A Z_B^2 Z_C}{(Z_A + Z_B + Z_C)^2}$$

$$Z_2 Z_3 = \frac{Z_B Z_C}{Z_A + Z_B + Z_C} \times \frac{Z_C Z_A}{Z_A + Z_B + Z_C} = \frac{Z_A Z_B Z_C^2}{(Z_A + Z_B + Z_C)^2}$$

$$Z_3 Z_1 = \frac{Z_C Z_A}{Z_A + Z_B + Z_C} \times \frac{Z_A Z_B}{Z_A + Z_B + Z_C} = \frac{Z_A^2 Z_B Z_C}{(Z_A + Z_B + Z_C)^2}$$

$$Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1 = \frac{Z_A Z_B^2 Z_C}{(Z_A + Z_B + Z_C)^2} + \frac{Z_A Z_B Z_C^2}{(Z_A + Z_B + Z_C)^2} + \frac{Z_A^2 Z_B Z_C}{(Z_A + Z_B + Z_C)^2}$$

$$= \frac{Z_A Z_B^2 Z_C + Z_A Z_B Z_C^2 + Z_A^2 Z_B Z_C}{(Z_A + Z_B + Z_C)} = \frac{Z_A Z_B Z_C (Z_A + Z_B + Z_C)}{(Z_A + Z_B + Z_C)^2}$$

$$= \frac{Z_A Z_B Z_C}{(Z_A + Z_B + Z_C)} \qquad ---(7)$$

Form eq.(7),

$$Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1 = \frac{Z_A Z_B Z_C}{(Z_A + Z_B + Z_C)} = Z_A \times \frac{Z_B Z_C}{(Z_A + Z_B + Z_C)} = Z_A Z_2 \qquad \left(\because \quad Z_2 = \frac{Z_B Z_C}{(Z_A + Z_B + Z_C)}\right)$$

$$\Rightarrow Z_A = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2} \qquad ---(8)$$

Form eq.(7),

$$Z_1Z_2 + Z_2Z_3 + Z_3Z_1 = \frac{Z_A\,Z_B\,Z_C}{(Z_A + Z_B + Z_C)} = Z_B \times \frac{Z_C\,Z_A}{(Z_A + Z_B + Z_C)} = Z_B\,Z_3 \qquad \left(\because \quad Z_3 = \frac{Z_C\,Z_A}{(Z_A + Z_B + Z_C)}\right)$$

$$\Rightarrow Z_B = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3} \qquad ---(9)$$

Form eq.(7),

$$Z_1Z_2 + Z_2Z_3 + Z_3Z_1 = \frac{Z_A\,Z_B\,Z_C}{(Z_A + Z_B + Z_C)} = Z_C \times \frac{Z_A\,Z_B}{(Z_A + Z_B + Z_C)} = Z_C\,Z_1 \qquad \left(\because \quad Z_1 = \frac{Z_A\,Z_B}{(Z_A + Z_B + Z_C)}\right)$$

$$\Rightarrow Z_C = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1} \qquad ---(10)$$

$$Z_A = \frac{Z_1\,Z_2 + Z_2\,Z_3 + Z_3\,Z_1}{Z_2}, \quad Z_B = \frac{Z_1\,Z_2 + Z_2\,Z_3 + Z_3\,Z_1}{Z_3} \;\;\&\;\; Z_C = \frac{Z_1\,Z_2 + Z_2\,Z_3 + Z_3\,Z_1}{Z_1}$$

Note: "The Delta element is equal to the sigma of product of all Star elements divided by corresponding opposite Star element".